

Trinucleon Electromagnetic Form Factors and the Light-Front Hamiltonian Dynamics

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Abstract. This contribution briefly illustrates preliminary calculations of the electromagnetic form factors of ${}^3\text{He}$ and ${}^3\text{H}$, obtained within the Light-front Relativistic Hamiltonian Dynamics, adopting i) a Poincaré covariant current operator, without dynamical two-body currents, and ii) realistic nuclear bound states with S , P and D waves. The kinematical region of few $(\text{GeV}/c)^2$, relevant for forthcoming TJLAB experiments, has been investigated, obtaining possible signatures of relativistic effects for $Q^2 > 2.5 (\text{GeV}/c)^2$.

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INTRODUCTION

Relativistic Hamiltonian Dynamics (RHD), introduced by Dirac in a seminal paper [1], represent a viable tool for fulfilling the Poincaré covariance, i.e. a fundamental symmetry. In particular, in the study of the electromagnetic (em) form factors (ff) of few-nucleon systems some advancements were made within the so-called Light-front (LF) RHD (one out the three ones proposed by Dirac), both calculating the Deuteron em observables [2] and obtaining a first description of the Trinucleon ff [3] (retaining only S and S' waves). It should be pointed out that relativistic calculations have a twofold interest, both from a general point of view and from a phenomenological one, given the forthcoming experiments at TJLAB, in the region of few $(\text{GeV}/c)^2$ (see, e.g., Ref. [4] for the ${}^3\text{He}$ and ${}^3\text{H}$ ff).

Our aim is to construct, within the LF RHD, a relativistic approach for light nuclei, taking into account only a fixed number of degrees of freedom, that i) automatically embeds the whole successful phenomenology already developed within the non relativistic framework, and ii) includes non perturbatively the relativistic features requested by the Poincaré covariance. The Bakamjian-Thomas (BT) procedure (see, e.g., Ref. [5]) allows us to implement the above program, since within such an approach it is possible to explicitly construct operators that fulfill the commutation rules of the Poincaré generators, in terms of i) operators that do not contain the interaction and ii) an operator that acts as the mass of the interacting system. This interacting mass operator, that depends upon intrinsic variables, has to satisfy the same constraints given by the Galilean group, namely the same properties implemented in the non relativistic quantum mechanics. In sum-

mary, one can exploit realistic wave functions for light nuclei (for $A=3$, see, e.g., Ref. [6]), that depend upon suitable Jacobi coordinates, in order to evaluate matrix elements of a Poincaré covariant current operator [7] for an interacting system. The appeal of the BT procedure, within the LF framework, is given by the fact that relativistic effects imposed by Poincaré covariance can be straightforwardly taken into account through the relativistic kinematics and the presence of the so-called Wigner-Melosh rotations (see, e.g., Ref. [5]), that ultimately lead to using the standard Clebsh-Gordan machinery for obtaining many-nucleon wave functions with the correct angular coupling.

FORMALISM

Among the main features of LF RHD (for an extended review see, e.g., Ref. [5]), one has to mention the largest number of kinematical Poincaré generators (seven, as dictated by the symmetry of the *initial* hypersurface $x^+ = 0$) and the simplest procedure for separating out the center of mass motion from the intrinsic one, in strict analogy with the non relativistic procedure (given the absence of the square root in the operator generating LF-time translation of the system). On one side, since the LF boosts form a subgroup of the kinematical set, it is not necessary any Wigner function for taking care of the boost transformation of the nuclear wave function, when we consider, e.g., a nuclear final state recoiling in the laboratory frame. On the other side, since the rotations around the x - and y -axes are dynamical ones, we need to overcome a difficulty. A possible strategy is suggested by the BT construction, that amounts to put in relation the LF spin with the canonical one through unitary transformations, namely Wigner-Melosh rotations. For the sake of concreteness, the Wigner-Melosh rotations in the 2×2 representation, are given by

$$\begin{aligned} D^{\frac{1}{2}}[\mathcal{R}_W(k_i^y)]_{\sigma\sigma'} &= \chi_{\sigma}^{\dagger} L_c^{-1}(k_i^y) L_{LF}(k_i^y) \chi_{\sigma'} = \\ &= \chi_{\sigma}^{\dagger} \frac{m + k_i^+ - \boldsymbol{\iota}\boldsymbol{\sigma} \cdot (\hat{\mathbf{z}} \times \mathbf{k}_{i\perp})}{\sqrt{(m + k_i^+)^2 + |\mathbf{k}_{i\perp}|^2}} \chi_{\sigma'} \end{aligned} \quad (1)$$

where $L_c^{-1}(k_i^y)$ and $L_{LF}(k_i^y)$ are the $\mathbf{SL}(2, \mathbf{C})$ representations of the canonical boost and the LF one, respectively (see, e.g. Ref. [7]), and k_i^y is the intrinsic momentum of the i -th constituent.

Therefore, the canonical spin of a single constituent (i.e., the total angular momentum in its intrinsic frame) is given in terms of the LF one, by

$$\vec{s}_c(i) = [\mathcal{R}_W(k_i^y)] \vec{s}_{LF}(i) \quad (2)$$

and the component of a state $|\psi\rangle$ in the LF-spin basis is related to its canonical counterpart, by

$${}_{LF}\langle\sigma|\psi\rangle = \sum_{\sigma'} D^{\frac{1}{2}}[\mathcal{R}_W^{\dagger}(k_i^y)]_{\sigma\sigma'} {}_c\langle\sigma'|\psi\rangle \quad (3)$$

This illustrates the source of a very cumbersome algebra necessary to perform Trinucleon calculations.

Following Ref. [7], for an interacting system, an em current operator, $J^\mu(0)$, that fulfills the extended Poincaré covariance (i.e. considering parity and time reversal, as well) and Hermiticity, can be constructed by a suitable auxiliary operator, j^μ , that fulfills rotational covariance around the z -axis in a Breit frame ($\mathbf{P}_f + \mathbf{P}_i = 0$), where the \perp component of the momentum transfer is vanishing ($\mathbf{q}_\perp = 0$). Note that such a frame is *different* from the Drell-Yan one, where $q^+ = 0$, where the kinematical symmetry around \hat{q} is not exploited. In general, the matrix elements $\langle P_f | J^\mu | P_i \rangle$, still acting on internal variables, are directly given by the matrix elements of the auxiliary operator j^μ , evaluated in the chosen Breit frame. A possible Ansatz for a *many-body* auxiliary operator is built from i) the free current (a one-body operator) and ii) the \perp component of the angular momentum operator \vec{S} (a many-body operator in LF) as follows

$$j_{fi}^\mu(q\hat{e}_z) = \frac{1}{2} \left[\mathcal{J}_{fi}^\mu(q\hat{e}_z) + L_v^\mu[r_x(-\pi)] e^{i\pi S_x} \mathcal{J}_{if}^\nu(q\hat{e}_z)^* e^{-i\pi S_x} \right] \quad (4)$$

where $r_x(\theta)$ is a rotation by an angle θ around the x -axis, $\mathcal{J}_{fi}^\mu(q\hat{e}_z) = \Pi_f J_{free}^\mu(0) \Pi_i$, with Π the projector onto the states of the (initial or final) system and $\vec{S} \equiv$ the LF-spin operator of the system as whole: it acts on the "internal" space. The operator, $J_{free}^\mu(0)$ is the proper sum over $A=2,3,\dots$ free Nucleon current given by

$$J_N^\mu = -F_2^N(\Delta^2) \frac{(p^\mu + p'^\mu)}{2M} + \gamma^\mu [F_1^N(\Delta^2) + F_2^N(\Delta^2)] \quad (5)$$

where $\Delta^2 = (p'^\mu - p^\mu)^2$ and $F_{1(2)}^N(\Delta^2)$ the Dirac (Pauli) Nucleon ff. It should be pointed out that the Nucleon ff depend upon $p'^\mu - p^\mu$ and not upon $q^\mu \neq p'^\mu - p^\mu$, since only three components of the four-momentum, i.e. p^+ and \mathbf{p}_\perp , are conserved quantities. On the other hand, within RHD framework all the particles are on their mass-shell.

In the chosen Breit frame, charge normalization and current conservation (for $M_f = M_i$) can be fulfilled by imposing $\mathcal{J}^-(q\hat{e}_z) = \mathcal{J}^+(q\hat{e}_z)$ [7, 2].

Summarizing, if $J^\mu(0)$, that is an operator acting in the whole space, is Poincaré covariant, then the intrinsic operator, j^μ , is invariant for rotations around the z -axis, and viceversa. Moreover, in Eq. (4), S_x generates many-body contributions to j^μ , as well as $\mathcal{J}_{fi}^\mu(q\hat{e}_z)$, if a many-body term is added to $J_{free}^\mu(0)$, as discussed in the following Section.

For evaluating matrix elements of $j^\mu(q\hat{e}_z)$, the eigenstates of the interacting system are needed. To this end, one can use the "non relativistic solutions", but with Wigner-Melosh rotations in the angular part, if the interaction $V \equiv M_{int} - M_0$ (where M_{int} is the mass operator of the interacting system and M_0 the corresponding free mass) can be embedded in a BT framework. The BT construction for obtaining interacting Poincaré generators suggests a necessary (not sufficient) condition on the interaction (see Ref. [5]): V must depend upon intrinsic variables combined in scalar products, i.e. $[\vec{\mathcal{B}}_{LF}, V] = [\vec{S}_0, V] = [P_\perp, V] = [P^+, V] = 0$, where $\vec{\mathcal{B}}_{LF}$ are the LF boosts, $\vec{S}_0 \equiv$ the angular momentum operator for the non interacting case (since $S_0^2 = S_{int}^2$ and $S_{0,z} = S_{int,z}$, the eigenvalues of S_0^2 and $S_{0,z}$ can label the eigenstates of the interacting system). The non relativistic interaction fulfills the above requirements.

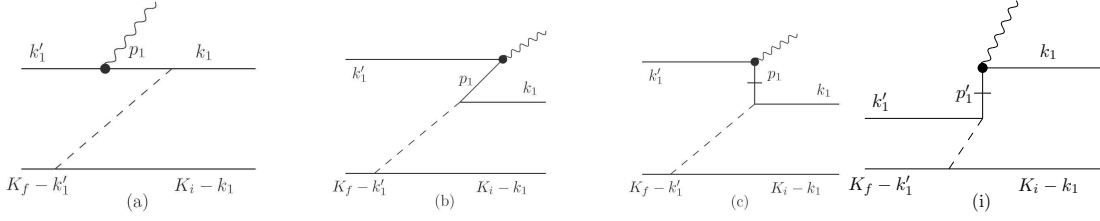


FIGURE 1. A class of two-body (at least three particles in flight) current contributions generated within a LF analysis of a Yukawa model, see text. Diagram (b) represents a pair production contribution. Diagrams (c) and (i) contain the instantaneous, in LF time, propagation of an internal fermionic particle, marked by a horizontal dash. The LF time flows from the right to the left. (After [8])

TRINUCLEON EM OBSERVABLES

The choice of a Breit frame where $\mathbf{q}_\perp = 0$, namely $q^+ \neq 0$, is a far reaching one, since, as above mentioned, one can find a simple constraint to be fulfilled by a one-body intrinsic current operator for recovering the Poincaré covariance, as well as by each many-body term that one could add to J_{free}^μ . Furthermore, this choice necessarily produces a two-body current related to a pair production [8] (see diagram (b) in Fig. 1), compelling us to consider the inclusion of a larger set of two-body currents in the future calculations. In particular, a recent analysis of a 4D Yukawa model [8], in ladder approximation, has led to a 3D current on the LF, fulfilling the Ward-Takashi Identity (for a general discussion see [8]). In Fig. (1), a set of contributions to the first-order (in the interaction) 3D LF current is shown. In the preliminary calculations of the Trinucleon em observables presented in this contribution, the two-body terms, like the ones depicted in Fig. (1), are not included, while the application to the Deuteron case is in progress. One can anticipate that i) the pair term affects all the three Deuteron ff, while instantaneous terms (present only for fermionic constituents) contribute to the magnetic one, ii) the pair term vanishes for $q^+ \rightarrow 0$, as it must do, while instantaneous ones survive, iii) the remaining, on-mass shell terms (like (a) in Fig. (1)) affect all the ff in the whole range of q^+ , iv) the pair term should be maximal at $q^+ \sim m_N$ (cf the discussion in [9]). The macroscopic current of the Trinucleon is given in terms of charge and magnetic ff by

$$J_{T_z}^\mu = -F_2^{T_z}(Q^2) \frac{(P^\mu + P'^\mu)}{2M_{T_z}} + \gamma^\mu (F_1^{T_z}(Q^2) + F_2^{T_z}(Q^2)) \quad (6)$$

where $T_z = \pm 1/2$ labels ${}^3\text{He}$ and ${}^3\text{H}$ respectively. Microscopic evaluations of the ff are based on proper traces of the current in Eq. (4), viz

$$\begin{aligned} F_{ch}^{T_z}(Q^2) &= F_1^{T_z} - \frac{Q^2}{4M_{T_z}^2} F_2^{T_z} = \frac{1}{2} \text{Tr}[j^+(T_z)] = \frac{1}{2} \text{Tr}[\mathcal{J}^+(T_z)] \\ F_{mag}^{T_z}(Q^2) &= F_1^{T_z} + F_2^{T_z} = -i \frac{M_{T_z}}{Q} \text{Tr}[\hat{\sigma}_y j_x(T_z)] = -i \frac{M_{T_z}}{Q} \text{Tr}[\hat{\sigma}_y \mathcal{J}_x(T_z)] \end{aligned} \quad (7)$$

where the matrix elements of the microscopic current are given by

$$\mathcal{J}_{\sigma'\sigma}^r(T_z) \equiv \langle \Psi_{\frac{1}{2}\sigma'}^{\frac{1}{2}T_z}, P' | \mathcal{J}^r | \Psi_{\frac{1}{2}\sigma}^{\frac{1}{2}T_z}, P \rangle \quad (8)$$

with $|\Psi_{\frac{1}{2}\sigma}\rangle$ the Trinucleon wave function. In the actual calculation we have used the bound states obtained through a variational technique [6] by taking into account two-nucleon forces, like *AV18* [10], and three-body ones, like *UIX* [11]. Moreover, ^3He and ^3H are distinct, since the Coulomb forces are included. The bound states contain S , S' , P and D waves, and the Wigner-Melosh rotations suitable for expressing the LF spins in terms of the standard ones. It should be pointed out that the numerical calculations involve 6D Montecarlo integrations.

In Tables 1 and 2, preliminary results for the static em properties of ^3He and ^3H are shown. In particular in the first Table, the bound states obtained by retaining only two-nucleon forces are presented, while in the second Table the bound states correspond to *AV18* + *UIX*. The benefits of considering the Poincaré covariance are clear and of the same order found in the Deuteron case [2]. We expect a possible improvement of the magnetic moments by including the instantaneous contributions (cf Fig. 1), since they particularly affect the magnetic ff only (as already seen for the Nucleon case [12]).

TABLE 1. Preliminary calculations of magnetic moments and charge radii of ^3He and ^3H . The two-body force, *AV18*, and the Coulomb interaction are included. Trinucleon wave functions from [6]. Probability of the waves considered: $\mathcal{P}_{S+S'}(\text{Av18}) \sim 91.4\%$, $\mathcal{P}_P(\text{Av18}) \sim 0.07\%$, $\mathcal{P}_D(\text{Av18}) \sim 8.5\%$

Theory	$\mu(^3\text{He})$	$\mu(^3\text{H})$	$r_{ch}(^3\text{He})\text{fm}$	$r_{ch}(^3\text{H})\text{fm}$
NR(S+S')	-1.700(1)	2.515(3)	1.926(3)	1.726(3)
LF(S+S')	-1.758(1)	2.600(3)	1.949(3)	1.771(3)
NR(S+S'+P+D)	-1.762(1)	2.579(2)	1.916(4)	1.718(4)
LF(S+S'+P+D)	-1.834(2)	2.674(2)	1.941(4)	1.759(4)
Exp.	-2.1276	2.9789	1.959(30)	1.755(86)

TABLE 2. Preliminary calculations of magnetic moments and charge radii of ^3He and ^3H . Two- and three-body forces, *AV18* + *UIX*, are included, as well as the Coulomb interaction. Trinucleon wave functions from [6]. Probability of the waves considered: $\mathcal{P}_{S+S'}(\text{Av18} + \text{UIX}) \sim 90.5\%$, $\mathcal{P}_P(\text{Av18} + \text{UIX}) \sim 0.01\%$, $\mathcal{P}_D(\text{Av18} + \text{UIX}) \sim 9.3\%$.

Theory	$\mu(^3\text{He})$	$\mu(^3\text{H})$	$r_{ch}(^3\text{He})\text{fm}$	$r_{ch}(^3\text{H})\text{fm}$
NR(S+S')	-1.697(1)	2.494(2)	1.848(3)	1.695(3)
LF(S+S')	-1.759(2)	2.588(2)	1.870(3)	1.712(3)
NR(S+S'+P+D)	-1.760(1)	2.569(2)	1.841(4)	1.666(4)
LF(S+S'+P+D)	-1.837(2)	2.669(2)	1.867(4)	1.690(4)
Exp.	-2.1276	2.9789	1.959(30)	1.755(86)

In the evaluation of the Trinucleon ff two different sets of Nucleon em ff have been considered: i) the Gari-Krümpelmann Nucleon ff [13] and ii) the ones obtained within a novel LF approach [12], see Figs. 2 and 3, in order to test the dependence of the Trinucleon ff upon the new features of the Nucleon ff, like a possible zero in the proton charge ff [12, 14].

The charge and magnetic ff of ^3H and ^3He , evaluated in the Breit frame where $\mathbf{q}_\perp = 0$ and with S , S' , P and D waves, are shown in Figs. 4 and 5 for bound states corresponding to two-nucleon forces, while in Figs. 6 and 7 the calculations with three-body forces

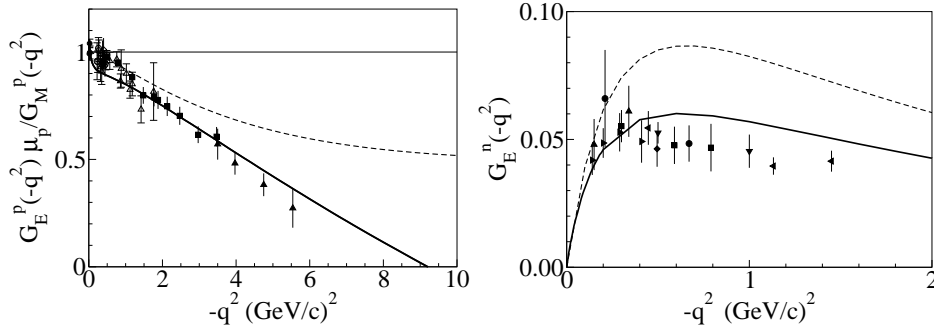


FIGURE 2. Left panel: $\mu^p G_E^p(Q^2)/G_M^p(Q^2)$. Right panel: charge neutron ff. Solid line: LF Nucleon ff [12]. Dashed line: Gari-Krümpelmann [13]

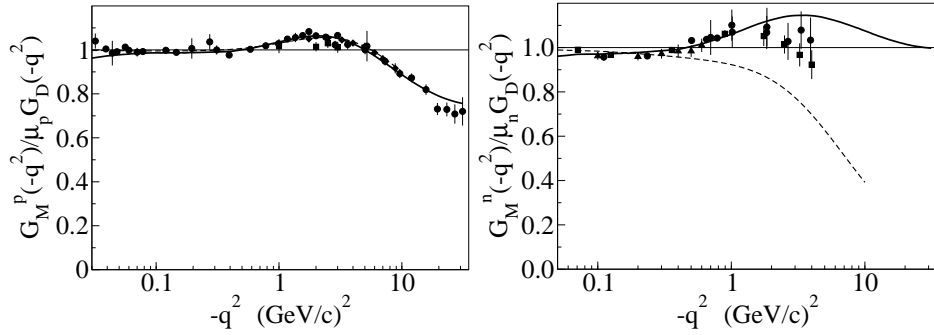


FIGURE 3. The same as in Fig. 2, but for the magnetic Nucleon ff.

are presented. Calculations with only $S + S'$ waves are shown in [3]. Some expected features, like the presence of large relativistic effects on the tails for $Q \geq 7$ ($1/fm$), are well confirmed, as well as the necessity of two-body dynamical corrections to the current operator (cf the positions of the minima). An interesting signature of the three-body forces can be found in the tails, since they give more binding and smaller charge radii. It could be relevant, if such an effect will still be present after including two-body dynamical currents. As a final remark, we should note that differences between relativistic calculations obtained by using the Gari-Krümpelmann ff and the ones calculated adopting the LF ff are not sizable.

CONCLUSIONS & PERSPECTIVES

In order to embed the Poincaré covariance in the description of light nuclei we adopt a Light-Front RHD and the Bakamjian-Thomas procedure. Extending our approach, already applied to the Deuteron case [2], the em observables of ^3He and ^3H have been calculated for the first time with all the waves, S , S' , P and D , in the bound states and taking into account the three-body forces as well. The relativistic effects on em observables at $Q^2 = 0$, though of the order of few % but in the correct direction, are encouraging. Moreover, the sizable effects at high Q^2 indicate the essential role played

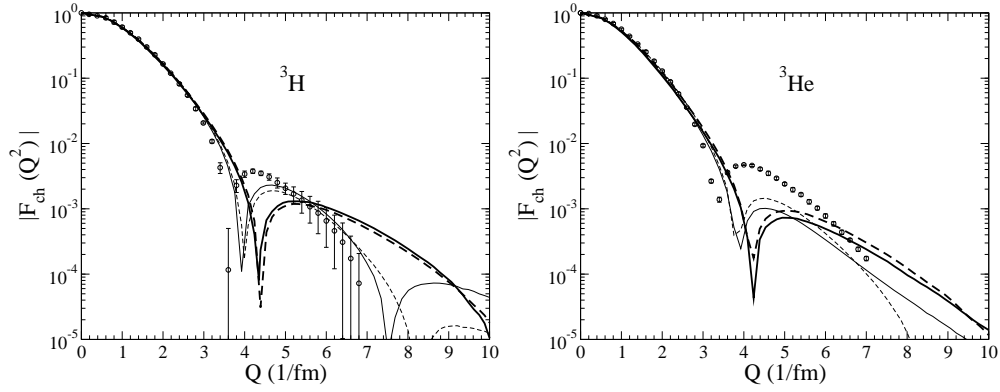


FIGURE 4. Preliminary calculations of ${}^3\text{H}$ and ${}^3\text{He}$ charge ff vs Q^2 . The adopted Trinucleon wave functions [6] contain both the AV18 two-nucleon interaction and the Coulomb interaction. $S + S' + P + D$ waves are taken into account. Thick lines: LF calculations. Solid line: full calculation and LF Nucleon ff [12]; dashed line: full calculation and Gari-Krümpelmann Nucleon ff [13]. Thin lines: non relativistic calculations. Data from [15].

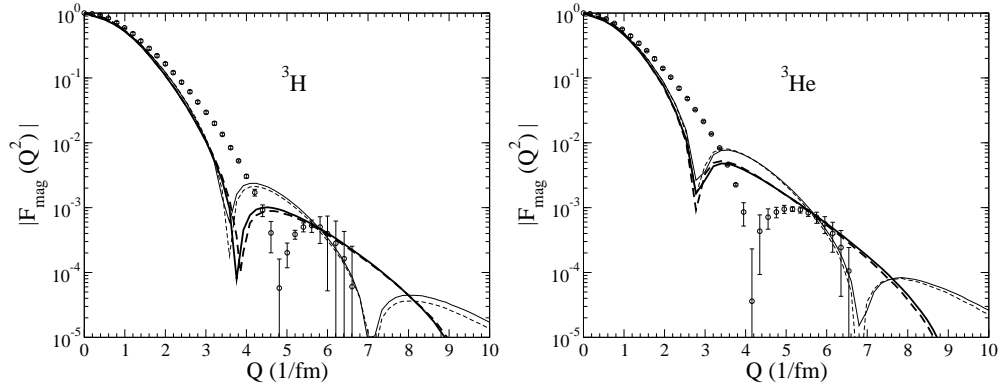


FIGURE 5. The same as in Fig. 4, but for the magnetic ff.

by the Poincaré covariance for analyzing the em ff in the region of few GeV's. Notably, three-body forces could be important in the same kinematical region.

A full calculation, with a systematic inclusion of two-body currents, like the ones shown in Fig. 1, will be presented elsewhere.

REFERENCES

1. P.A.M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
2. F. M. Lev, E. Pace and G. Salmè, Phys. Rev. **C 62**, 064004 (2000) and Phys. Rev. Lett. **83**, 5250 (1999).
3. F. A. Baroncini, E. Pace and G. Salmè, to be published in Few-Body Systems and arXiv:0712.0516.
4. J. R. Arrington et al, "Elastic Electron Scattering off ${}^3\text{He}$ and ${}^4\text{He}$ at Large Momentum Transfers", E04-018, TJLAB.
5. B.D. Keister and W. Polyzou, Adv. Nucl. Phys. **20**, 1 (1991).
6. A. Kievsky, M. Viviani, and S. Rosati, Nucl. Phys. **A 577**, 511 (1994).
7. F. M. Lev, E. Pace and G. Salmè, Nucl. Phys. **A 641**, 229 (1998).
8. A. O. Marinho, T. Frederico, E. Pace, G. Salmè and P. U. Sauer, Phys. Rev. **D 77** (2008) 116010.

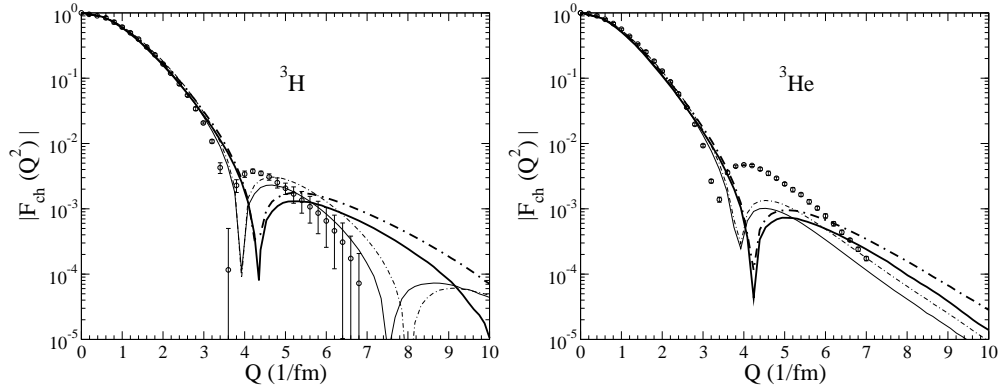


FIGURE 6. Preliminary calculations of ${}^3\text{H}$ and ${}^3\text{He}$ charge ff vs Q^2 . The adopted Trinucleon wave functions [6] contain both i) two-nucleon and three-body forces: AV18+UIX and ii) Coulomb interaction. $S + S' + P + D$ waves are taken into account. Thick lines: LF calculations. Solid line: full calculation with the AV18 two-nucleon interaction & LF Nucleon ff [12]. Dash-dotted line: calculation with two-nucleon+three-body interactions (AV18+UIX) & LF Nucleon ff. Thin lines: non relativistic calculations. Data from [15].

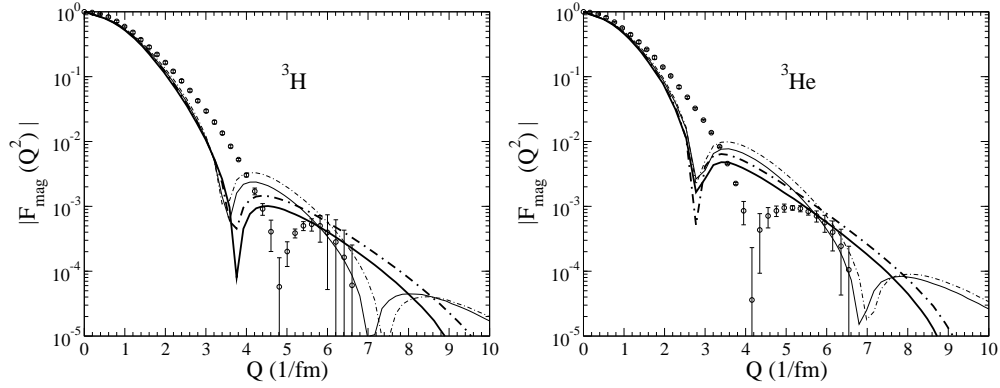


FIGURE 7. The same as in Fig. 6, but for the magnetic ff.

9. J.P.B.C. de Melo , T. Frederico, E. Pace , and G. Salmè, Nucl. Phys. **A 707**, 399 (2002).
10. R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. **C 51**, 38 (1995).
11. B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, Phys. Rev. **C 56**, 1720 (1997).
12. J.P.B.C. de Melo , T. Frederico, E. Pace , S. Pisano , G. Salmè, arXiv:0804.1511.
13. M. Gari and W. Krümpelmann, Z. Phys. **322**, 689 (1985).
14. M.K. Jones et al., Phys. Rev. Lett. **84**, 1398 (2000); O. Gajou et al., Phys. Rev. Lett. **88**, 092301 (2002); V. Punjabi et al., Phys. Rev. **C 71**, 055202 (2005) and erratum, ibidem, 069902.
15. I. Sick, Prog. Part. Nucl. Phys. **47**, 245 (2001) and Refs. therein quoted.